CHIZIQLI DIFFERENSIAL TENGLAMANI YECHISHNING BESH XIL USUL VA SHUNINGDEK, MATEMATIK PAKETLAR IMKONIYATLARIDAN FOYDALANISH MATEMATIKA O‘QITISH METODIKASIDA METODIK TAHLIL SIFATIDA.

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Annotatsiya: Ushbu maqolada chiziqli differensial tenglamalarni yechishning besh xil usuli ilmiy-metodik jihatdan tahlil qilingan. Shuningdek, MATLAB, Maple, Mathematica va Python (SymPy) kabi zamonaviy matematik paketlarning imkoniyatlari hamda ulardan matematika ta’limi jarayonida samarali foydalanish masalalari yoritilgan. Har bir usulning afzalliklari, qo‘llanish sohasi va talabalarning mantiqiy hamda matematik tafakkurini rivojlantirishdagi o‘rni ko‘rsatib berilgan.

Kalit so‘zlar: chiziqli differensial tenglama, integrallovchi ko‘paytuvchi, o‘zgarmaslarni variatsiyalash usuli, xarakteristik tenglama, Laplas almashtirishi, MATLAB, Maple, Mathematica, SymPy.

FIVE METHODS FOR SOLVING LINEAR DIFFERENTIAL EQUATIONS AND THE USE OF MATHEMATICAL SOFTWARE CAPABILITIES AS A METHODOLOGICAL ANALYSIS IN MATHEMATICS TEACHING METHODOLOGY.

Abstract: This article presents a scientific and methodological analysis of five different methods for solving linear differential equations. It also examines the capabilities of modern mathematical software packages—such as MATLAB, Maple, Mathematica, and Python (SymPy)—and discusses their effective use in mathematics education. The advantages and application areas of each method are outlined, along with their role in developing students' logical and mathematical thinking.

Keywords: linear differential equation, integrating factor, method of variation of constants, characteristic equation, Laplace transform, MATLAB, Maple, Mathematica, SymPy.

ПЯТЬ МЕТОДОВ РЕШЕНИЯ ЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ И ИСПОЛЬЗОВАНИЕ ВОЗМОЖНОСТЕЙ МАТЕМАТИЧЕСКОГО ПРОГРАММНОГО

ОБЕСПЕЧЕНИЯ КАК ЭЛЕМЕНТ МЕТОДОЛОГИЧЕСКОГО АНАЛИЗА В МЕТОДИКЕ ПРЕПОДАВАНИЯ МАТЕМАТИКИ.

Аннотация: В статье представлен научно-методический анализ пяти различных методов решения линейных дифференциальных уравнений. Также рассматриваются возможности современных пакетов математического программного обеспечения — таких как MATLAB, Maple, Mathematica и Python (SymPy) — и обсуждаются вопросы их эффективного использования в математическом образовании. Охарактеризованы преимущества и области применения каждого метода, а также их роль в развитии логического и математического мышления студентов.

Ключевые слова: линейное дифференциальное уравнение, интегрирующий множитель, метод вариации постоянных, характеристическое уравнение, преобразование Лапласа, MATLAB, Maple, Mathematica, SymPy.

Kirish. Differensial tenglamalar matematikaning muhim bo‘limlaridan biri bo‘lib, fizika, texnika, iqtisodiyot, biologiya va boshqa ko‘plab fanlarda uchraydigan jarayonlarni matematik modellashtirishda keng qo‘llaniladi. Shu sababli oliy ta’lim muassasalarida differensial tenglamalarni o‘qitish alohida ahamiyat kasb etadi. Zamonaviy ta’lim jarayonida an’anaviy yechish usullarini axborot texnologiyalari bilan uyg‘unlashtirish o‘quv jarayonining samaradorligini oshiradi.

Asosiy qism.(MR)

1**. Integrallovchi ko‘paytuvchi usuli**

Integrallovchi ko‘paytuvchi usuli — bu to‘la differensialga keltirilmaydigan M(x,y)dx + N(x,y)dy = 0 ko‘rinishdagi birinchi tartibli differensial tenglamani mos μ(x,y) funksiyaga ko‘paytirish orqali to‘la differensial tenglamaga aylantirib yechish usulidir.

M(x,y) dx +N(x,y)dy = 0

Agar $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ bo‘lsa, tenglama to‘la differensial bo‘lmaydi.

Integrallovchi ko‘paytuvchi : Shunday μ(x,y) funksiya topiladiki, uni tenglamaga ko‘paytirganda μMdx+ μNdy=0 to‘la differensial tenglamaga aylanadi.

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x}$$

- Faqat x ga bog’liq bo‘lsa (μ= μ(x)):
  $\frac{1}{M}(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) = g(y)$
  bo‘lsa, $\mu(x) = e^{\int f(x)dx}$ bo‘ladi.

- Faqat y ga bog’liq bo‘lsa (μ= μ(y)):

$\frac{1}{M}(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) = g(y)$

bo‘lsa , μ(y)= $e^{\int g(y)dy}$ bo‘ladi.

Namuna uchun misol(A):

$(y+xy^2)dy - xdy = 0$

$M(x,y) = y+xy^2$

$N(x,y) = -x$

$\frac{\partial M}{\partial y} = 1 + 2xy$

$\frac{\partial N}{\partial y} = -1$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ bo‘lgani uchun bu tenglama to‘la differensial emas . Integrallovchi ko‘paytuvchini (μ) topamiz. Faqat y ga bog’liq integrallovchi ko‘paytuvchi shartini tekshiramiz .

$\frac{1}{M}(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) = \frac{1}{y+xy^2}(-1-(1+2xy)) = \frac{-2-2xy}{y(1-xy)} = -\frac{2}{y}$

$g(y) = -\frac{2}{y}$

μ(y)= $e^{\int g(y)dy} = e^{\int -\frac{2}{y}dy} = e^{-2\ln(y)} = \frac{1}{y^2}$

μ(y)= $\frac{1}{y^2}$

$\frac{y+xy^2}{y^2}dx - \frac{x}{y^2}dy = 0$

$(\frac{1}{y}+x)dx - \frac{x}{y^2}dy = 0$

$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2}$

$\frac{\partial N_1}{\partial x} = -\frac{1}{y^2}$ tenglama to‘la differensial bo‘ldi.

Shunday U(x,y) = c funksiya borki , uning differensiali yangi tenglamaga teng .

U(x,y)= $\int M_1 dx = \int (\frac{1}{y} + x)dx = \frac{x}{y} + \frac{x^2}{2} + \varphi(y)$

$$\frac{\partial U}{\partial y} = -\frac{x}{y^2} + \dot{\varphi}(y) = -\frac{x}{y^2}$$

$$\dot{\varphi}(y) = 0 \quad \varphi(y) = \text{const.}$$

Yakuniy javob : $\frac{x}{y} + \frac{x^2}{2} = c$

2. O‘zgaruvchini almashtirish usuli

O‘zgaruvchini almashtirish usuli — bu differensial tenglamani soddaroq yoki ma'lum bir standart ko‘rinishga (masalan, o‘zgaruvchilari ajraladigan tenglamaga) keltirish uchun yangi yordamchi o‘zgaruvchi kiritish usulidir.

Bu usul ko‘pincha quyidagi ikki holatda ko‘p qo‘llaniladi:

1. y' = f(ax + by + c) ko‘rinishdagi tenglamalarda (z = ax + by + c almashtirish olinadi).
2. $y = \grave{f}'(\frac{y}{x})$ ko‘rinishidagi bir jinsli tenglamalarda (u=$\frac{y}{x}$ yoki ux) almashtirish olinadi.

Namuna uchun misol (A):

$y' = (x+y)^2$

$z = x+y$
$z' = 1+ y'$
$y' = z' - 1$
$z' - 1 = z^2$
$z' = z^2 + 1$
$z' = \frac{dz}{dx}$

$$\frac{dz}{dx} = z^2 + 1$$

$$\int \frac{dz}{z^2+1} = \int dx$$

Arctg(z) = x+c

Dastlabki z =x +y almashtirishni qaytarib joyiga qo‘yamiz.

arctg (x+y) = x+c

x+y = tg(x+c)

Yakuniy javob : y = tg(x +c) - x

3. O‘zgarmaslarni variatsiyalash usuli

O‘zgarmaslarni variatsiyalash usuli (yoki Lagranj usuli) — bu chiziqli bir jinsli bo‘lmagan differensial tenglamalarni yechishning universal usulidir.

Bu usulning mohiyati shundaki, avval tenglamaning o‘ng tomoni nolga tenglashtirilib, bir jinsli qismi yechiladi. So‘ngra, topilgan yechimdagi o‘zgarmas son C o‘rniga noma'lum funksiya C(x) olingan deb qaraladi va u topiladi.

Eng ko‘p tarqalgan birinchi tartibli chiziqli tenglama misolida buni ko‘rib chiqamiz.

Namuna uchun misol(A) :

$y' - \frac{y}{x} = x^2$

$y' - \frac{y}{x} = 0$

$y' = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y}{x}$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln|y| = \ln|x + \ln|c||$

$y_0 = c\ x$

o‘zgarmaslarni variantsiyalaymiz

$y = c(x)\ x$

$y' = c'(x)\ x + c(x)$

Asl tenglamaga qo‘yib, c(x) ni topamiz .

$(c'(x)\ x + c(x)) - \frac{c(x)x}{x} = x^2$

$c'(x)\ x + c(x) - c(x) = x^2$

$c'(x)\ x = x^2$

$c'(x) = x$

$c(x) = \int x dx = \frac{x^2}{2} + c$

(Bu yerda c -haqiqiy ixtiyoriy o‘zgarmas son)

y=($\frac{x^2}{2}$ + c_1) x

Yakuniy javob : $y = \frac{x^3}{2} + c_1$ x

4. Xarakteristik tenglama usuli

Xarakteristik tenglama usuli — bu o‘zgarmas koeffitsiyentli chiziqli bir jinsli differensial tenglamalarni yechishning eng sodda va samarali usulidir. Usulning mohiyati differensial tenglamani oddiy algebraik tenglamaga (xarakteristik tenglamaga) keltirishga asoslangan.

Eng ko‘p uchraydigan ikkinchi tartibli o‘zgarmas koeffitsiyentli tenglama misolida ko‘rib chiqamiz

Usulning qoidasi: Differensial tenglamadagi hosilalar mos ravishda algebraik darajalarga almashtiriladi:

- y'' → k^2
- y' → k
- y → 1 (ya'ni k^0)

Xarakteristik tenglamaning ildizlariga (k_1, k_2) qarab umumiy yechim shakli tanlanadi:

1. Ildizlar haqiqiy va har xil ($k_1 \neq k_2$)

   $y=c_1e^{k_1x} + c_2e^{kx_2}$

2. Ildizlar haqiqiy va teng ($k_1= k_2=k$)

   $y=(c_1+c_2x)e^{kx}$

3. Usulning kompleksli ko‘rinishi:

$y = e^{2x}(C_1\cos x + C_2\sin x)$

$y = e^{2x}(C_1\cos\beta x + C_2\sin\beta x)$

Namuna uchun misol(A):

1) y'' - 5y' + 6y = 0

$k^2 - 5k + 6 = 0$

$k_1 = 2, k_2 = 3$

Yakuniy javob : $y = C_1e^{2x} + C_2e^{3x}$

2) y'' - 4y' + 4y = 0

$k^2 - 4k + 4 = 0$

$(k - 2)^2 = 0$

$k_1 = k_2 = 2$

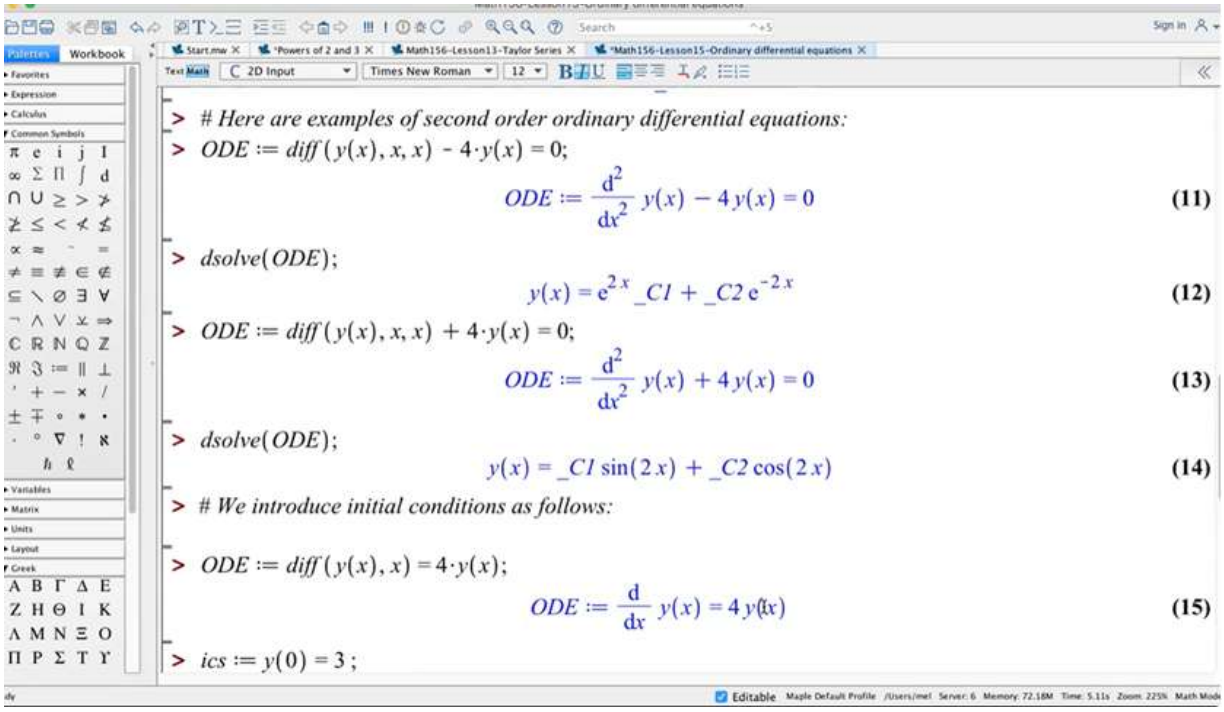
Yakuniy javob :$y = (C_1 + C_2x)e^{2x}$

5. Laplas almashtirishi usuli

Laplas almashtirishi usuli — bu differensial tenglamalarni (ayniqsa, boshlang‘ich shartlar berilgan Koshi masalalarini) yechishning operatorlar usulidir. Bu usul yordamida murakkab differensial amallar oddiy algebraik tenglamalarga keltiriladi.

Matematik paketlardan foydalanish

Bugungi kunda matematik paketlardan foydalanish ta’lim sifatini oshirishda muhim omillardan biri hisoblanadi. MATLAB – differensial tenglamalarni sonli va analitik yechish, grafiklar chizish hamda modellashtirish imkoniyatiga ega. Maple – murakkab matematik ifodalarni soddalashtirish va analitik yechimlarni olish uchun qulay dastur. Mathematica – yuqori aniqlikdagi hisoblashlar, matematik modellashtirish va vizual tahlilni amalga oshiradi. Python (SymPy) – bepul va ochiq kodli platforma bo‘lib, differensial tenglamalarni ramziy usulda yechish imkonini beradi. Mazkur dasturlardan foydalanish talabalarga yechimlarni tekshirish, grafiklarni tahlil qilish hamda murakkab hisoblashlarni tez va aniq bajarishga yordam beradi.



Xulosa. Chiziqli differensial tenglamalarni o‘qitishda avvalo analitik yechish usullarini puxta o‘rgatish maqsadga muvofiqdir. Shundan so‘ng matematik paketlar yordamida olingan natijalarni tekshirish va tahlil qilish tavsiya etiladi. Bunday yondashuv talabalar bilimini mustahkamlaydi, mustaqil fikrlash ko‘nikmalarini rivojlantiradi hamda zamonaviy raqamli texnologiyalardan samarali foydalanish malakasini shakllantiradi.Chiziqli differensial tenglamalarni yechishning besh xil usulini taqqoslab o‘qitish talabalar tomonidan mavzuni chuqurroq o‘zlashtirishga xizmat qiladi. Zamonaviy matematik paketlardan foydalanish esa nazariy bilimlarni amaliyot bilan bog‘lash, hisoblash aniqligini oshirish va o‘quv jarayonining samaradorligini yuksaltirishga imkon yaratadi.

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